# DEFORMATION OF A FLOWING POWDER WITH NONVISCOUS FRICTION

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UDC 624.131.54

A model for an ideally plastic body is extended to a powder medium with nonorthogonal slip lines; it is shown that some properties of the ideal-plasticity model are not essential and should not be generalized, namely, the coincidence between the characteristics for the velocity and stress distributions, and independence of the velocity distribution from the possible differences in the shears on the areas. A closed system of equations is derived, and the properties of the discontinuous solutions are discussed; boundary-value problems are formulated. It is shown that the stability-loss lines are arcs of circles (surface of a circular cylinder) in the case of bank stability.

1. Consider the planar deformation of an unconsolidated medium in the limiting state:

$$\tau = \sin\varphi\sigma + k \tag{1.1}$$

where $\sigma$ and $\tau$ are the invariants of the stress tensor, while $\varphi$ and k are constants of the material. On the area $\alpha$ (where $\alpha$ is the angle between the area and the largest compressive stress $\sigma_1$) the tangential and normal stresses $\tau_\alpha$ and $\sigma_\alpha$ are related by

$$\tau_\alpha = c_1(\alpha)\,\sigma_\alpha + c_2(\alpha)$$
$$c_1 = \frac{\sin\varphi\sin 2\alpha}{1-\sin\varphi\cos 2\alpha}, \quad c_2 = \frac{k\sin 2\alpha}{1-\sin\varphi\cos 2\alpha} \tag{1.2}$$

The structure of (1.2) reflects the law of friction between the particles: if $\tau_\alpha$ and $\sigma_\alpha$ satisfy (1.2) for certain critical coefficients $\pm c_1^\circ$ and $\pm c_2^\circ$, then relative slip between the particles is possible; if $c_1(\alpha)$ and $c_2(\alpha)$ do not equal the critical values, the contacts between the particles are below the limiting state, and the deformation on the area remains stable. Let $c_1(\alpha) = c_1^\circ$, $c_2(\alpha) = c_2^\circ$ for $\alpha = \alpha_0 > 0$, where $\alpha_0$ is a known constant. The values of $\alpha_0$ may be dependent on the properties of the material and on the loading conditions below the limiting state [1, 2].

Then in the limiting state of (1.1) the deformation mechanism is anisotropic: on areas tangential to the lines $x_2'(x_1) = \mathrm{tg}(\theta \pm \alpha_0)$ one can have unbounded relative displacements between particles, while on all other areas the displacements between particles are small [3] ($\theta$ is the inclination of $\sigma_1$ to the $Ox_1$ axis, while $Ox_1x_2$ is a Cartesian coordinate system). Such a deformation mechanism may be interpreted as follows: in the limiting state, the medium is divided up by lines $x_2' = \mathrm{tg}(\theta \pm \alpha_0)$ into regular elements, and subsequent deformation occurs by rotation, compression, and slip of the elements, one with respect to the other. To construct a closed system of equations to describe the deformation one needs to formulate the requirements that the system has to meet; only one specification is necessary in this case [4]: the system must describe in invariant form the processes that occur on the areas $x_2' = \mathrm{tg}(\theta \pm \alpha_0)$. Possible additional conditions are not necessary, including the condition that the characteristics for the velocity and stress fields coincide.

The deformation kinetics will be dependent on the law of friction between the elements; there are two essentially different modes of frictions: viscous and normal [3] or nonviscous. In viscous friction, the shear rate between elements is dependent on nonlocal factors (behavior or neighboring elements and the

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Novosibirsk. Translated from Zhurnal Prikladnoi Matematiki i Tekhnicheskoi Fiziki, No. 4, pp. 119-124, July-August, 1974. Original article submitted February 15, 1974.

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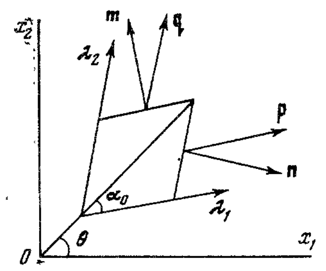

Fig. 1

boundary conditions) and on local factors (stresses acting on the slip area). In the nonviscous case, the friction is dependent on the stresses only up to the onset of slip; subsequently, the velocity is not dependent on stresses and can be restricted only by nonlocal factors. In a hardening plastic solid, one gets the first case, whereas the second occurs in an ideally plastic body. In what follows we restrict ourselves to a model for an unconsolidated medium with viscous friction. The distribution of $\sigma$ and $\theta$ will be assumed fixed and known from the solution of the boundary-value problem for the stresses [5].

2. Let $v_1(x_1x_2)$ and $v_2(x_1x_2)$ be the velocity distribution in the Cartesian coordinate system; by $\partial$ we denote the increments in the velocities and coordinates on passing from element to element, while by $\delta$ we denote the increments within an element. The shear velocities on the areas AB and BC (Fig. 1) may be characterized via the invariant quantities

$$\gamma_{qp} = \frac{\partial v_p}{\partial q} + \omega \sin 2\alpha_0 = \frac{\cos 2\theta}{2}\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) + \frac{\sin 2\theta}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) + \frac{\cos 2\alpha_0}{2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) - \frac{\sin 2\alpha_0}{2}\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) + \omega \sin 2\alpha_0 \tag{2.1}$$

$$\delta_{pq} = \frac{2v_q}{\partial p} - \omega \sin 2\alpha_0 = \frac{\cos 2\theta}{2}\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) + \frac{\sin 2\theta}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) + \frac{\cos 2\alpha_0}{2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) + \frac{\sin 2\alpha_0}{2}\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) - \omega \sin 2\alpha_0$$

where $\omega = {}^1/_2\,(\delta v_2/\delta x_1 - \delta v_1/\delta x_2)$, as slip on the nonorthogonal areas AB and BC does not cause any volume change; the change must be due to the sum $\partial v_n/\partial p + \partial v_m/\partial q$, and this becomes

$$\frac{\partial v_n}{\partial p} + \frac{\partial v_m}{\partial q} = \sin 2\alpha_0\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) \tag{2.2}$$

If we assume that the velocity components $v_n$ and $v_m$ are continuous together with their derivatives, we get the estimate

$$\frac{\partial v_n}{\partial p} + \frac{\partial v_m}{\partial q} = \sin 2\alpha_0\left(\frac{\delta v_1}{\delta x_1} + \frac{\delta v_2}{\delta x_2}\right) + o(l)_n \tag{2.3}$$

where $l$ is the distance between slip lines. Such estimates are impossible for (2.1), because the discontinuity at the side of the elements in the tangential velocities is in essence due to the deformation, as is the possible independence of the shear velocities $\gamma_{qp}$, and $\gamma_{pq}$, then $\omega$ must be left as an independent function for $l \to 0$. The increments $\partial$ in that case may be considered as differentials, which enables us to describe the behavior of a discontinuous medium by means of the mechanics of continuous media for small distances between the lines of discontinuity.

Let $(\lambda_1, \lambda_2)$ be the natural parameters of the slip lines $x_2' = \mathrm{tg}(\theta \pm \alpha_0)$, while $w_1$ and $w_2$ are the projections of the velocity on the normals to the sides of an element. As the friction has been assumed nonviscous, the velocity of an element $(\lambda_1, \lambda_2)$ may be bounded only by the velocities of elements $(\lambda_1 \pm d\lambda_1, \lambda_2 \pm d\lambda_2)$, and the condition for continuity in $v_n$ and $v_m$ goes with the condition for possible slip on the sides of an element to imply that only information about the velocity $w_1$ normal to the side of an element can be transmitted along the line $\lambda_1$:

$$\begin{gathered} \bar{\mathbf{v}}(\lambda_1 + d\lambda_1)\cdot\bar{\mathbf{n}}(\lambda_1) = w_1(\lambda_1) - \omega(\lambda_1)\cos 2\alpha_0 - \varepsilon_1(\lambda_1) \\ \frac{\partial w_1}{\partial \lambda_1} - \frac{w_2 + \cos 2\alpha_0 w_1}{\sin 2\alpha_0}\frac{\partial \theta}{\partial \lambda_1} + w \cos 2\alpha_0 + \varepsilon_1 = 0 \end{gathered} \tag{2.4}$$

where $\varepsilon_1$ is the compression rate of the element along the line $\lambda_1$. Similarly, along $\lambda_2$

$$\frac{\partial w_2}{\partial \lambda_2} + \frac{w_1 + \cos 2\alpha_0 w_2}{\sin 2\alpha_0}\frac{\partial \theta}{\partial \lambda_2} - \omega \cos 2\alpha_0 + \varepsilon_2 = 0 \tag{2.5}$$

It follows from (2.4) and (2.5) that the velocity distribution is, in general, dependent on the distribution of $\omega$, which reflects the possible differences in the slip area functioning. An exception must be made for an ideally plastic medium (ideally coupled [5]), for which $\alpha_0 = \pi/4$ and for which (2.4) and (2.5) are closed no matter what the equation for $\omega$. It can be shown that the condition for the stress- and strain-rate tensors to be coaxial in the ideal plasticity case is equivalent to the condition for symmetry in compressibility: $\varepsilon_1 = \varepsilon_2$; if then $\varepsilon \equiv \varepsilon_1 + \varepsilon_2 \equiv 0$, then (2.4) and (2.5) become equations for an incompressible ideally plastic medium [6].

Let $\varepsilon$ be known as a function of the hydrostatic pressure, while $\Delta = \varepsilon_1/\varepsilon_2$ is either low from additional considerations or is equal to 1; (2.5) reflects the possibility of slip from the line $x_2' = \operatorname{tg}(\theta \pm \alpha_0)$, but the line $x_2' = \operatorname{tg}(\theta + \alpha_0)$ will be a characteristic of the system for $\omega_1$, $\omega_2$, and $\omega$ only if the equation for $\omega$ contains no derivatives along $\lambda_1$. If this is not so, weak discontinuities in the velocity will propagate along a line different from the slip line It follows from (2.5) that locally a line of weak discontinuity consists of parts $x_2' = \operatorname{tg}(\theta + \alpha_0)$, but the rotation of the elements causes the propagation direction for the weak discontinuities to deviate from the slip line. An ideally plastic material is an exception in that respect. If $\alpha_0 = \pi/4$ we have $\omega \cos 2\alpha_0 \equiv 0$, and the transfer of the slip line due to rotation occurs along the slip line itself. Then the characteristics of the velocity distribution for an ideally plastic material coincide with slip lines (if these are defined as in Sec. 1) and with the characteristics of the stress distribution.

Consider the equations needed to close (2.4) and (2.5); if none of the slip lines is specially distinguished in the loading history of material below the limit or in the boundary conditions for the limiting state, we can assume that the shear rates $\gamma_{qp}$ and $\gamma_{pq}$ average over a certain time interval are equal. In that case $\omega = {}^1/_2(\partial v_2/\partial x_1 - \partial v_1/\partial x_2)$ and the characteristics of the velocity distribution coincide with the lines of largest tangential stress $x_2' = \operatorname{tg}(\theta \pm \pi/4)$; if here $\Delta = 1$, the stress- and strain-rate tensors will be coaxial.

We now consider another limiting case where the slip occurs along one of the lines $\lambda_1$ and $\lambda_2$, for instance $\lambda_1$. Then $\gamma_{pq} \equiv 0$ and the complete system of equations takes the form

$$\sin 2\theta\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) - \cos 2\theta\left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}\right) - \cos 2\alpha_0\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) + \sin 2\alpha_0\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) + 2\omega\cos 2\alpha_0 + 2\varepsilon_1 = 0 \qquad (2.6)$$

$$-\sin 2\theta\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) + \cos 2\theta\left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}\right) + \cos 2\alpha_0\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) + \sin 2\alpha_0\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) - 2\omega\cos 2\alpha_0 + 2\varepsilon_2 = 0$$

$$\cos 2\theta\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) + \sin 2\theta\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) + \cos 2\alpha_0\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) + \sin 2\alpha_0\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) - 2\omega\sin 2\alpha_0 = 0 \qquad (2.7)$$

System (2.6) and (2.7) is of hyperbolic type. If $\mu_1$ and $\mu_2$ are the natural parameters of the characteristics $x_0' = \operatorname{lg}(\theta - \alpha_0)$ and $x_2' = -\operatorname{ctg}(\theta - \alpha_0)$ for the system, with $u_1$, $u_2$ the projections of the velocity characteristics, while $f_1$ and $f_2$ are the projections of the velocity on the slip line $\lambda_2$ and the path orthogonal to this, than (2.6) and (2.7) may be put at

$$\begin{aligned} \frac{\partial u_1}{\partial \mu_1} - u_2\frac{\partial \theta}{\partial \mu_1} &= -\varepsilon_1 \sin 2\alpha_0 \\ \frac{\partial u_2}{\partial \mu_2} + u_1\frac{\partial \theta}{\partial \mu_2} &= -\frac{\varepsilon_1\cos^2 2\alpha_0 + \varepsilon_2}{\sin 2\alpha_0} \\ \frac{\partial f_1}{\partial \mu_1} - f_2\frac{\partial \theta}{\partial \mu_1} &= \omega \sin 2\alpha_0 \end{aligned} \qquad (2.8)$$

The formulation of the boundary-value problems is determined by the system (2.6) and (2.7); if both velocity components are specified along the boundary $x_2 = x_2(x_1)$, $\nu = \operatorname{arc\,tg} x'_2$ the formulation will be correct if the boundary does not have characteristic directions for $\mu_1$ and $\mu_2$. The boundary conditions should satisfy the first two relations in (2.8) along the characteristics. Let $\omega$ be given as the boundary. Only some combination of the partial derivatives with respect to the velocity may be specified at the boundary in addition to this condition, for instance $P\beta$:

$$\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_2}x_2'\right)\cos\beta + \left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_2}{\partial x_2}x_2'\right)\sin\beta = \frac{1}{\cos\nu}P_\beta$$

The correctness or otherwise is dependent not only on (2.6) and (2.7) but also on the form of the boundary condition for the velocity; the formulation will be correct if

$$x_2' \neq \frac{\cos(\theta + \alpha_0 - \beta)}{\cos(\theta - \alpha_0 - \beta)}\operatorname{tg}(\theta - \alpha_0)$$

with the boundary conditions related as follows for $x_0' = \operatorname{tg}(\theta - \alpha_0)$:

$$\omega\sin(\theta - \alpha_0 - \beta) + \varepsilon_1\sin(\theta + \alpha_0 - \beta) = -P_\beta$$

If $\beta = \theta - \alpha_0 + \pi/2$ we must have as follows for any position of the boundary

$$\omega\cos(\theta - \alpha_0 - \nu)\sin 2\alpha_0 + \varepsilon_1\cos 2\alpha_0\sin(\theta + \alpha_0 - \nu) + \varepsilon_2\sin(\theta - \alpha_0 - \nu) = \sin 2\alpha_0 P_\beta$$

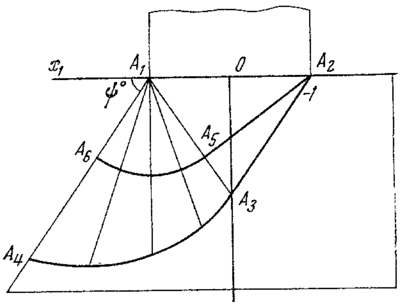


Fig. 2

Then the Cauchy, Goursat, and various mixed problems will be correct for system (2.6) and (2.7).

The velocities appear in (2.6) and (2.7) in differential form, while $\omega$ appears algebraically, one has to consider weak discontinuities in the velocity and strong discontinuity in $\omega$ together with discussing the discontinuous solution. Let $\varepsilon_1$, $\varepsilon_2$, and $\theta$ be continuous; weak discontinuities in the velocities are possible only on the characteristics $x_2' = \mathrm{tg}(\theta - \alpha_0)$, $x_2' = -\mathrm{ctg}(\theta - \alpha_0)$. The roles of the characteristics will then be different: a weak velocity discontinuity occurs on a slip line while $\omega$ remains continuous, while on a line orthogonal to the slip line a weak discontinuity in the velocity involves a strong discontinuity in $\omega$. Analogous conclusions apply for strong velocity discontinuities and for the integral of infinite discontinuity in $\omega$. If the stresses are discontinuous on a certain curve, then the velocities will also be discontinuous. The discontinuities will satisfy (2.6) and (2.7) in the sense of generalized solutions [7].

The case of slip on $\lambda_2$ is reduced to the following by replacing $\alpha_0$ by $-\alpha_0$.

3. We consider as an example the stability of a sloping bank bounded by the smooth press $A_1A_2$ (Fig. 2). To $A_1A_2$, and $A_1A_4$ there are applied loads such that $\sigma = \sigma^\circ = \mathrm{const}$ on $A_1A_2$, $\sigma = \sigma^\circ \exp 2\mathrm{tg}\, \varphi \times (\psi^\circ - \alpha_0 - \pi/2)$ on $A_1A_4$, where $\alpha_0 = \pi/4 - \varphi/2$, $\alpha_0 < \psi^\circ < \pi/2$; in that case, a continuous stress distribution can be constructed in the region $A_1A_2A_3A_4$ [5]: $\sigma \equiv \sigma^\circ$, $\theta \equiv \pi/2$, in $A_1A_2A_3$, and $\sigma = \sigma^\circ \exp 2\mathrm{tg}\, \varphi\, (\psi - \alpha_0 - \pi/2)$ $\theta = \psi - \alpha_0$ in $A_1A_3A_4$, where $(r, \psi)$ is a polar coordinate system. We assume that the loss of stability means that the press tips over with a certain angular velocity $\Omega > 0$; $v_2 = \Omega(1 - x_1)$, $x_2 = 0$, $|x_1| \le 1$, and we also assume that in region $A_1A_2A_3A_4$ only the areas of the family $x_2' = \mathrm{tg}(\theta + \alpha_0)$ are ready to slip, while the adhesion conditions are met at the boundary $A_2A_3A_4$. As the line $A_2A_3A_4$ is not a characteristic of the velocity distribution, the conditions on $A_2A_3A_4$ imply that the region $A_2A_3A_4A_6A_5$ will remain immobile. On line $A_2A_5A_6$ the condition for continuity in the normal velocity is met, the solution in the regions $A_1A_2A_5$ and $A_1A_5A_6$ t takes the following form: $(\varepsilon_1 \equiv \varepsilon_2 \equiv 0)$

$$\begin{aligned} v_1 &= \Omega \,\mathrm{ctg}\, \alpha_0 \,(1 - x_1 + \mathrm{ctg}\, \alpha_0 x_2) \\ v_2 &= \Omega\,(1 - x_1 + \mathrm{ctg}\, \alpha_0 x_2) \\ v_r &\equiv 0; \\ v_\psi &\equiv \frac{\Omega}{\sin^2 \alpha_0} r \end{aligned} \tag{3.1}$$

It follows from (3.1) that strong discontinuity in the tangential velocity can be realized only on the surface $A_5A_6$ of a circular cylinder and the area $A_2A_5$.

4. In closing (2.4) and (2.6) it was assumed that the slip occurs along the family of $\lambda_1$, lines. We can consider the case of alternating slip on both possible families $\lambda_1$ and $\lambda_2$, in which case the velocities $v_1$, $v_2$, and $\omega$ are split up into two components, each of which satisfies equations of the form (2.6) and (2.7). The division of the boundary conditions into components may then not be unique, but the ambiguity is removed by additional data for detailed cases; it may be that a solution exists for certain boundary conditions only for a certain devision by components; when one is seeking boundary conditions that provide a trivial velocity distribution, it is sufficient to consider all possible styles of slip, and so on.

System (2.6) and (2.7) is unaltered if we assume that the medium is dilated in accordance with some present law; in that case $\varepsilon_1$ and $\varepsilon_2$ will be dependent on the corresponding shear.

5. We consider now the interpretation of (2.6); it can be shown that the contribution from side AB (Fig. 1) to the volume change in an element is represented by the invariant quantity

$$\begin{aligned} \frac{\partial v_m}{\partial q} - \omega \cos 2\alpha_0 = &-\frac{\sin 2\theta}{2}\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) + \frac{\cos 2\theta}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) + \\ &+ \frac{\sin 2\alpha_0}{2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) + \frac{\cos 2\alpha_0}{2}\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}\right) - \omega \cos 2\alpha_0 \end{aligned} \tag{5.1}$$

Similarly for side C

$$\begin{aligned} \frac{\partial v_n}{\partial p} + \omega \cos 2\alpha_0 = &\frac{\sin 2\theta}{2}\left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_2}{\partial x_2}\right) - \frac{\cos 2\theta}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) + \\ &+ \frac{\sin 2\alpha_0}{2}(\partial v_1 / \partial x_1 + \partial v_2 / \partial x_2) - \cos 2\alpha_0 / 2\,(\partial v_2 / \partial x_1 \\ &- \partial v_1 / \partial x_2) + \omega \cos 2\alpha_0 \end{aligned} \tag{5.2}$$

We equate $-\varepsilon_2$ and $-\varepsilon_1$ to the right sides of (5.1) and (5.2) to get (2.6); in that interpretation, equations (2.6) allow generalization to the case of the axially symmetric total limiting states [8].

See [9] for a detailed bibliography for the various models involving slip on limiting lines.

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